

## THE REALIZATION OF THE SECOND

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### INTRODUCTION

A primary cesium beam frequency standard serves to realize the unit of time, the Second, in accordance with the international definition as formulated at the XIII General Conference of Weights and Measures in 1967. The basic design of a cesium standard is shown in Fig. 1. The cesium beam emerges from an oven into a vacuum, passes a first state selecting magnet, traverses a Ramsey type cavity where it interacts with a microwave signal derived from a slave oscillator. The microwave signal changes the distribution of states in the atomic beam which is then analyzed and detected by means of the second state selector magnet and the atom detector. The detector signal is used in a feedback loop to automatically keep the slave oscillator tuned. The line-Q is determined by the interaction time between the atoms and the microwave cavity. Thus a beam of slow atoms and a long cavity leads to a high line-Q. Commercial devices which for obvious reasons are restricted in total size have line-Q's of a few  $10^7$ , whereas high performance laboratory standards with an overall device length of up to 6-m feature line-Q's of up to  $3 \times 10^8$ .

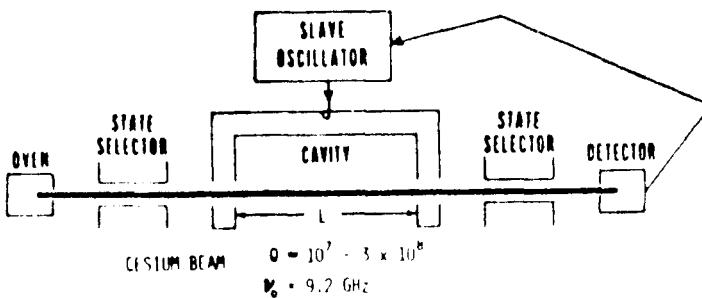


Fig. 1. Schematic of a Cesium Beam Standard

The National Bureau of Standards has two primary standards for the unit of time. They are both cesium devices and are designated NBS-4 and NBS-6. NBS-4 and NBS-5 (predecessor of NBS-6) have been used since January 1973 for a total of 22 calibrations of the NBS Atomic Time Scale. The independently evaluated accuracies (1) for NBS-4 and NBS-5 are  $3.1 \times 10^{-13}$  and  $1.8 \times 10^{-13}$ , respectively. NBS-5 was removed from service in February 1974. Major revisions in its oven/detector and vacuum system were carried out and the new system is now designated NBS-6. It has been operating since February 1975 and preliminary data indicate that the accuracy of NBS-6 will exceed the above quoted values.

#### ACCURACY OF A TIME SCALE

The accuracy of the rate (frequency) of an atomic time scale is the degree to which its Second agrees with the definition of the SI-second. Primary frequency standards are used to calibrate the rate of an atomic time scale. If these calibration errors are independent from one calibration to any other, and if the calibrations could be compared perfectly in time, then the error of the average would reduce as  $n^{-\frac{1}{2}}$ , where  $n$  is the number of calibrations. However, the calibrations cannot be averaged perfectly: there exists no perfect reference; also, for a given primary standard and even for a set of primary standards the errors of one calibration may well be correlated (in space or time) with some other calibration. A more general model for the errors involved in any given calibration (say the  $\ell$ th calibration) may be written (2):

$$\sigma_s^2(\ell) = \sigma_{ruc}^2(\ell) + \sigma_{\overline{ruc}}^2(\ell) \quad (1)$$

where  $\sigma_s(\ell)$  is the overall accuracy for the calibration,  $\sigma_{ruc}(\ell)$  is an estimate of the random uncorrelated errors and  $\sigma_{\overline{ruc}}(\ell)$  is an estimate of errors that are correlated with some of the past calibrations or with some other primary standard due to similarity of design or evaluation procedure (the bar over "ruc" denotes the logical "not").

An accuracy algorithm can be developed which incorporates the contributions of a series of calibrations by a primary standard. Based on comparisons via International Atomic Time (TAI) we find that NBS-4, NBS-5 and the two other operating primary standards at the Physikalisch Technische Bundesanstalt in Germany and the National Research Council in Canada agree within about  $1 \times 10^{-13}$  (3). Therefore we are assessing  $\sigma_{\overline{ruc}} = 0.5 \times 10^{-13}$  (2) and obtain through an accuracy algorithm the filtered calibrations against  $AT_0(NBS)$  shown in Fig. 2 (the lower dashed line in Fig. 2 connects the unfiltered original calibration points). The frequency  $AT(NBS)$  which is derived from an ensemble of commercial cesium standards has not been changed as a result of any of the listed calibration data, but has been maintained, as nearly as possible, as an independent continuing stable frequency reference. Also shown in Fig. 2 are the  $\frac{1}{2}$  year average

frequencies of TAI with respect to the NBS best estimate of cesium frequency obtained from the above mentioned accuracy algorithm. The comparisons were made via Loran-C data taken at the end of February and of August (BIH Circular D). The  $+1.8 \times 10^{-13}$  gravitational "blue shift" at Boulder has been accounted for in the data. We would conclude, therefore, that the TAI second is too short by  $8 \pm 2$  parts in  $10^{13}$  at the beginning of 1975. In the fall of 1973 the NBS primary standards and the German and Canadian primary standards agreed that the frequency of TAI was about  $10 \times 10^{-13}$  too high, i.e., the length of the TAI second was too short by this amount (3). It may be anticipated that future, regular input from all available primary standards will be incorporated into a TAI accuracy algorithm which would keep the TAI second bounded to within at least  $1 \times 10^{-13}$  of the best realization of the SI-Second. This would significantly decrease the time errors in TAI with regard to ideal "absolute" time and correspondingly increase the utility of TAI for fundamental physical and astrophysical measurements of extended duration.

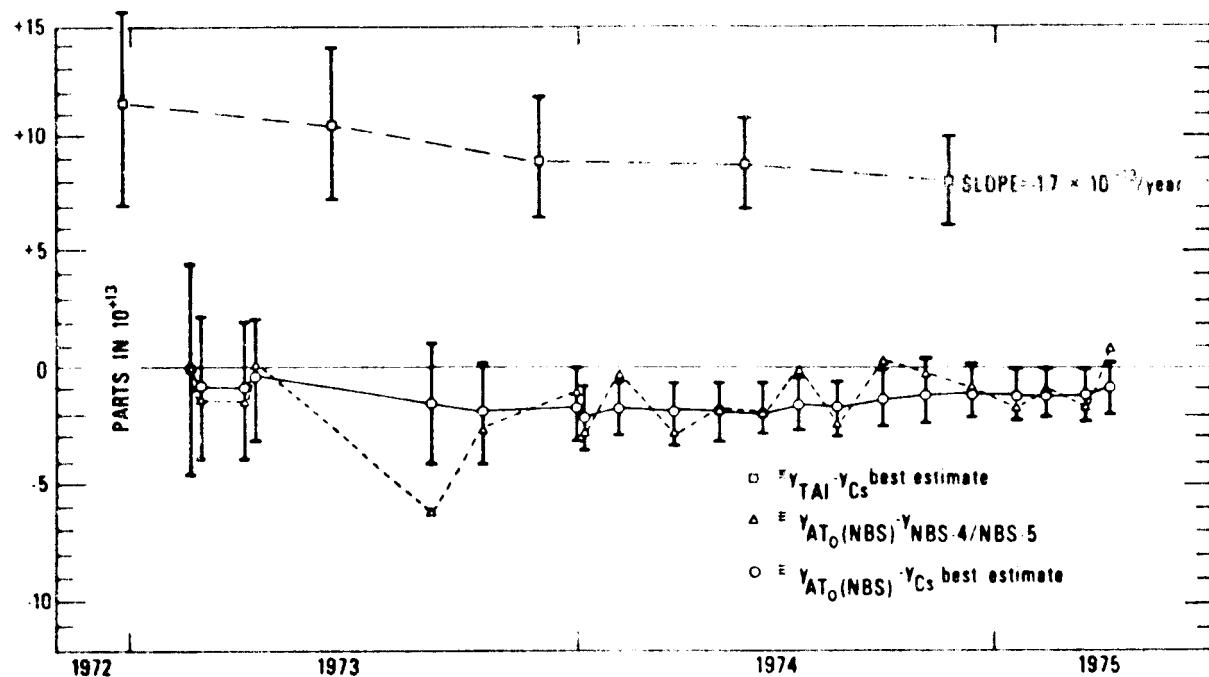


Fig. 2. Plot of the Rates of TAI and  $\Delta T_0$  (NBS) With Respect to the NBS Primary Standards from November 1973 to the Present

#### STABILITY LIMITATIONS

A prerequisite for achieving high accuracy is high stability. The ultimate stability limitation of all frequency standards, including cesium, is properly described by a process with a flicker of frequency spectral density (flicker floor). The physical causes for this noise are not yet well understood; however, a better understanding and corresponding improvements in the flicker floor are a prerequisite for significant increases in accuracy. The following is an attempt to briefly sketch out likely physical mechanisms for stability limitations.

(a) magnetic fields.

From the well known relationship (4) for the frequency shift in a magnetic field  $H$  in cesium we calculate that a field change  $\Delta H = 0.18 \mu G$  causes a fractional frequency change  $\Delta v/v = 1 \times 10^{-15}$  if a nominal C-field on  $H = 60 \text{ mG}$  is applied in the standard. Because the applied C-field will always be substantially larger than the residual magnetic fields in the shielded region, we need not be concerned about magnetic field changes in directions other than the applied C-field. In a controlled laboratory environment, we may find changes of the environmental field of at most  $10^{-3} \text{ G}$ . Thus, in order to assure  $10^{-15}$  frequency stability we need shielding factors of almost  $10^6$ . This should not be difficult to achieve; however, this is only a valid statement for beam tubes with C-fields applied in a radial direction as in most tubes including all NBS tubes. If a cavity configuration is chosen where the C-field must be applied in the axial direction, the effectiveness of the shielding is reduced in relation to the length/diameter ratio  $L/D$ . In long beam tubes this could become critical since this geometric effect (5) reduces the longitudinal shielding factor, for example, by more than a factor of 10 as compared to the shielding of the radial component in the case of  $L/D = 10$ , and will approach zero for  $L/D \rightarrow \infty$ .

Of special concern, beyond the nominal shielding factor, may be the stability in time of the residual magnetization of the shields. Little is known about such effects. We have some experimental evidence suggesting that time-varying mechanical stress may cause field changes in NBS-6 (4 m length of the shield package). We measure field changes of the order of a few hertz (a fraction of a hertz in NBS-4) in the Zeeman frequency over days which is equivalent to a stability of about  $10^{-14}$  in the clock frequency; this is the flicker noise level observed in stability comparisons between NBS-4 and NBS-6 and, before, between NBS-4 and NBS-5.

(b) oven temperature.

The oven temperature acts via the velocity dependent effects: cavity phase shift caused by a phase difference  $\delta$  between the two sections of the cavity, and the second-order Doppler effect: The frequency shift due to velocity dependent effects can be written:

$$\Delta v = - V_p \frac{\delta}{2\pi L} - V_D^2 \frac{\delta}{2c^2} \quad (2)$$

where  $L$  is the separation between the two cavity sections,  $c$  is the speed of light,  $v_0$  is the cesium resonance frequency and  $V_p$  and  $V_D$  are some mean velocities of the atomic beam (6). Generally, the beam optics strongly influences the mean velocity. However, as a worst condition we may assume that a temperature fluctuation  $\delta T$  of the cesium oven directly influences the mean velocity via

$$\frac{\delta V}{V} = \frac{\delta T}{2T} \quad (3)$$

The mean velocity in cesium tubes is of the order of 100 m/s and oven temperatures of about 360°K are typical. The fundamental frequency bias due to velocity dependent effects rarely exceed a value of  $10^{-11}$ . For these conditions we calculate a needed oven temperature stabil-

ity of slightly better than  $0.1^\circ\text{K}$  for  $10^{-15}$  fractional frequency stability. This is well within technical possibilities.

(c) cavity temperature variations.

A temperature difference  $\Delta T$  between the two cavity arms causes a change in  $L$  of  $\Delta L = \alpha L \Delta T$ .  $\alpha$  is the coefficient of thermal expansion. This causes a change in the cavity phase difference of

$$\delta(\delta) = \Delta L (\partial\delta/\partial x) \quad (4)$$

where  $(\partial\delta/\partial x)$  is the maximum phase gradient across the cavity window. and relates to the cavity  $Q$  ( $\partial\delta/\partial x = 0$  in an ideal, lossless cavity). Eq. 4 combined with the phase term in Eq. (2) yields

$$\delta\Delta v/v = (\alpha V_p / 2\pi v) (\partial\delta/\partial x) \Delta T \quad (5)$$

which interestingly, is independent of  $L$ . We calculate for a copper cavity, estimating  $\partial\delta/\partial x < 3 \times 10^{-4} \text{ rad/cm}$  (9); a needed temperature difference stability of 0.5 degrees to assure a frequency stability of  $10^{-15}$ .

(d) microwave power.

The effective mean velocities  $V_p$  and  $V_D$  in Eq. (2) are dependent on the interrogating microwave power. This is because the transition probability has the following proportionality (6).

$$P \sim \sin^2 2 b\tau \quad (6)$$

where  $b^2$  is proportional to the microwave power, and  $\tau$  is the transit time for an atom in one cavity section. If the velocity distribution is known,  $V_p$  and  $V_D$  can be calculated exactly (7). This, in fact, has been used for the determination of  $\delta$  in accuracy evaluations of cesium standards at the National Bureau of Standards (1). In the presence of a non-vanishing cavity phase difference  $\delta$ , fluctuations of the microwave power will cause frequency fluctuations. In order to estimate the sensitivity we simplify Eq. (2) by setting  $V_p = V_D$  and obtain

$$\frac{\delta\Delta v}{v} = \left[ \frac{V}{c^2} + \left( \frac{\Delta v}{v} \right)_p \frac{1}{V} \right] \delta V \quad (7)$$

where  $(\Delta v/v)_p$  corresponds to the phase shift term in Eq. (2).

For  $V = 100 \text{ m/s}$  and  $(\Delta v/v)_p = 10^{-11}$  we obtain  $\delta V \approx 10^{-2} \text{ m/s}$  in order to limit  $\delta\Delta v/v$  to  $10^{-15}$ . This is a very stringent requirement; from typical velocity distributions (1,6) it may be estimated that this corresponds to a microwave power stability of about  $10^{-2} \text{ dB}$ . Of course, smaller values for  $(\Delta v/v)_p$  or  $\delta$  are realizable and thus relax this requirement.

#### FUNDAMENTAL ACCURACY LIMITATIONS

Unfortunately, we cannot assume that  $\delta$  is a unique value for a given cavity and fully determined by its geometry. We have to consider that the cavity is lossy and that openings for passage of the beam are of finite size. This leads to the concept of distributed cavity phase variations which make  $\delta$  dependent on the particular tra-

jectory location in the cavity (8). All known accuracy evaluation techniques for the determination of  $\delta$  affect the atomic trajectories, whether it is beam reversal, power shifts (via the velocity dispersive nature of the beam optics), etc. (3). Therefore, a full determination of  $\delta$  and the associated frequency bias is an elusive goal, especially since a modeling of the distributed phase variations in the cavity appears not to be practical. The effect obviously is reduced by the choice of long cavities; e.g., we estimated a fundamental limitation in the determination of  $\delta$  for NBS-5, i.e., its accuracy, of somewhat better than an equivalent non-evaluatable frequency bias of  $1 \times 10^{-13}$  (8). The question arises of how to increase further the available accuracy under this condition. If no radical departure from a conventional beam tube design is considered, then the following design recommendations could be made: Geometrically narrow beams with stable trajectories, (comp. next section for NBS-6) reduction of velocity dispersive character of the beam optics, and better controlled cavities with reduced electrical losses. Narrow cylindrical or sheet type beams could be obtained by hexapole or special dipole optics, respectively. A more fundamental approach appears possible: significantly slower atomic velocities. This would reduce all velocity dependent effects correspondingly and would have the added benefit of a spectrally narrower atomic resonance. This approach is experimentally a challenge; however, it appears to be the only one promising substantial improvement. If slow beams of adequate intensity could be realized, then accuracy values of much better than  $10^{-13}$  appear possible. Cesium may or may not be the atom of choice in this case.

#### ACCURATE CLOCKS

It is not only desirable to increase the accuracy of the primary standard, but also to make use of this accuracy. The present requirement of interrupting the operation of the standard in order to evaluate its full accuracy is deficient in that it disallows the use of the standard's superior stability performance for the generation of time. Conversely, the use of the primary standard as a clock leads to a deterioration of its realized accuracy because some parameters affecting its frequency may slowly change, unnoticed.

An analysis of the effects which are most likely to be slowly time dependent and which have significant impact on the accuracy shows the following possibilities:

(1) microwave power. The power could be monitored independent of the beam tube and corrected (by a servo) if necessary.

(2) magnetic field. The field can be measured via the magnetic field dependent transitions ( $m_F \neq 0$ ). A measurement during clock operation could be executed by interrogating symmetrically with respect to the  $m_F = 0$  transition. The regular servo could easily discriminate against such additional (likely, much slower) modulation. This technique could be realized with no effect on the frequency other than a small deterioration of the shot-noise limited stability of the standard. Magnetic field inhomogeneities, however, may limit the usefulness of this approach.

(3) cavity phase difference.

Here, advantage may be taken of a spatial velocity dispersion of the beam optics which forces different velocities on different trajectories. In the deflection plane of dipole optics, we thus have a velocity scale, and different detector positions will detect different mean velocities. Two detectors installed at an offset from each other in the direction of deflection essentially examine two beams with different mean velocities. Simultaneous use of these detectors will give information on changes in the cavity phase difference. For example, one detector may be used for the main frequency servo, the other to generate an error signal which depends on  $\delta$  (comp. Eq. (2)). This information could be used continually to apply bias corrections or to drive a servo system. Other uses are possible such as the creation of a mixed signal which is in first-order independent of  $\delta$ . We have installed such a detector in NBS-6 in order to establish the feasibility of such a technique. NBS-6 with its on-line beam geometry making use of both atomic levels in two separate beams (1) would also allow the use of the dual detector to optimize trajectory symmetry and thus to evaluate more accurately the distributed cavity phase (see above). Thus, NBS-6 may surpass significantly the accuracy of NBS-5.

All of the above measures are no full substitute for a true accuracy evaluation; however, once such an evaluation is executed these techniques would maintain this accuracy in the operating standard; their use would allow a long-term clock operation of the primary standard with no sacrifice in the realized accuracy.

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